Question 1

(a) Consider the following argument.

> If bears are brown then rabbits are not red. If giraffes are not green then rabbits are red. Giraffes are green. Therefore bears are brown.

Define the following propositions:

b: bears are brown
g: giraffes are green
r: rabbits are red

(i) Rewrite the argument in symbols.
(ii) Construct a truth table for each premise and the conclusion.
(iii) State whether or not the argument is valid, and explain your answer.

(b) The propositions below can be arranged in three groups so that each member of a group is logically equivalent to the other two. Find the three groups.

1 If the system is not ready then the light is not on.
2 Either the system is not ready or the light is not on.
3 If the system is ready then the light is on.
4 Either the system is not ready or the light is on.
5 If the light is on then the system is ready.
6 If the light is on then the system is not ready.
7 If the system is ready then the light is not on.
8 Either the light is not on or the system is ready.
9 If the light is not on then the system is not ready.

Give your answer by listing the three groups, for example "2, 5 and 9" (which is not necessarily a correct answer)

[Hint: It may help to realise that the first three of the expressions

$$p \to q, \quad \sim q \to \sim p, \quad \sim p \vee q, \quad q \to p$$

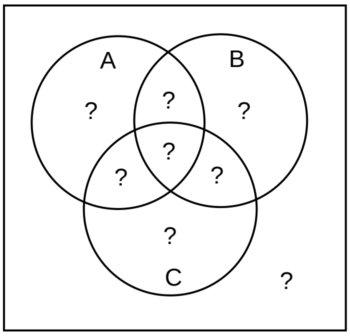
are logically equivalent to each other but they are not logically equivalent to the fourth (as you could verify using a truth table).]

[5 + 3 = 8 marks]

Question 2

A group of 140 students is polled to see how many watched three TV shows, Awesome, Bongo and Crikey. The results showed that 66 watched Awesome, 63 watched Bongo, 63 watched Crikey, 34 watched Awesome and Bongo, 31 watched Awesome and Crikey, 26 watched Bongo and Crikey, and 27 did not watch any of the three. Let A denote the set of students who watched Awesome, and similarly define sets B and C.

(a) Calculate the number of students in each of the eight subsets shown in the Venn diagram. Copy the Venn diagram and enter the number of students in each subset.



(b) Hence find how many students watched:
  (i) Awesome and Crikey, but not Bongo;
  (ii) Bongo only;
  (iii) only two of the three shows;
  (iv) at least two of the shows.

[8 + 2 = 10 marks]

Question 3

Suppose that P is a set of people and M a set of movies. Define the predicate S(p, m) to mean "person p has seen movie m". Consider the following list of sentences.

1 Everybody has seen all of the movies.
2 There is some person who has not seen any of the movies.
3 There is some movie that nobody has seen.
4 There is at least one movie that everybody has seen.
5 Nobody has seen any of the movies.
6 There is at least one movie that some person has not seen.
7 There is some person who has seen all the movies.
8 For each movie there is somebody who has not seen it.
9 Everybody has seen at least one movie.
10 For each person there is some movie that they have not seen.
11 For each movie there is somebody who has seen it.
12 There is at least one movie that some person has seen.

For each sentence, the negation is also in the list. Your task is to match each sentence with its negation. Give your answer by listing the number of each sentence and its negation, for example "4 and 10" (which is not necessarily a correct answer.)

[6 marks]

Question 4

Introduction:

There is a simple way of representing sets using so-called "bit strings". A bit string is simply a string of 0's and 1's. Suppose that U is the universal set. Then the rule for the bit string for a particular set S is:

> If the k^{th} element of U is in S, the k^{th} bit of the bit string is 1.
> If the k^{th} element of U is not in S, the k^{th} bit of the bit string is 0.

(Usually the order of the elements of a set is irrelevant. However when using bit strings the order of the elements as specified in the universal set U must be adhered to.)

Example:

Suppose the universal set is U = {1, 2, 3, 4, …, 10}. Then the set A = {1, 3, 5, 9} is represented by the bit string 1010100010 because:

the 1^{st} element of U **is** in A, so the 1^{st} element of the bit string is 1;
the 2^{nd} element of U is **not** in A, so the 2^{nd} element of the bit string is 0, and so on.

Similarly, the set {5, 6, 7} is represented by the bit string 0000111000.

Conversely, the bit string 0110010001 represents the set C = {2, 3, 6, 10} because:

the 1^{st} element of the bit string is 0, so the 1^{st} element of U, that is 1, is **not** in C;
the 2^{nd} element of the bit string is 1, so the 2^{nd} element of U, that is 2, **is** in C;
and so on.

Similarly, the bit string 0000000110 represents {8, 9}, and the bit string 0101010101 represents {2, 4, 6, 8, 10}.

The operations of union, intersection and complement may be carried out on the bit strings. The simplest way is to use a table and to work "bitwise", that is carry out the operations on the first bits, then the second bits, and so on.

Example:

Let the universal set be U = {4, 6, 9, 13, 18, 25}, and consider the sets A = { 6, 13, 18 } and B = {4, 13, 18, 25}. Suppose that we want to find $A \cup B$, $A \cap B$ and A'. Using a table and performing the operations "bitwise":

U	4	6	9	13	18	25
A	0	1	0	1	1	0
B	1	0	0	1	1	1
$A \cup B$	1	1	0	1	1	1
$A \cap B$	0	0	0	1	1	0
A'	1	0	1	0	0	1

Then, from the table,

$A \cup B$ is represented by: `110111`, so $A \cup B$ = {4, 6,13, 18, 25}
$A \cap B$ is represented by: `000110`, so $A \cap B$ = {13, 18}
A' is represented by: `101001`, so A' = {4, 9, 25}

Your task:

Let the universal set U be the following set of 16 countries:

U = {Angola, Benin, Chad, Djibouti, Eritrea, Ghana, Kenya, Libya, Mali, Namibia, Rwanda, Sudan, Tanzania, Uganda, Zaire, Zambia}

(i) Find the bit string that represents the set L = { Benin, Chad, Libya, Tanzania }

(ii) Find the set M represented by the bit string 0010 1100 0011 0001

(iii) Let P, Q, R and S be sets represented by the following four bit strings respectively:

P	0	1	1	1	0	1	0	1	0	1	1	0	1	0	0	1
Q	0	1	0	1	0	0	1	1	1	0	1	0	1	0	1	1
R	1	1	0	1	1	1	0	0	0	1	0	0	1	1	1	0
S	1	1	0	1	0	1	1	0	1	0	0	1	1	0	0	1

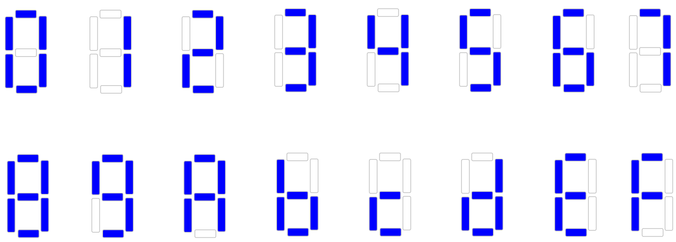
By using bitwise operations on the four bit strings, find the bit string that represents the set T = $(P' \cap Q) \cup (R \cap S')$. **(Set out your working in a table.)**

(iv) List the countries in the set T defined in (iii).

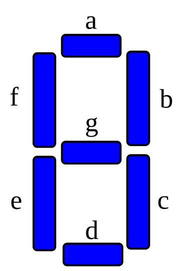
[1+ 1 + 3 + 1 = 6 marks]

Question 5

In the lecture notes there is an application of Karnaugh maps to a 7-segment display that is used in some calculators and similar digital devices. The display is sometimes **extended** to include **all** hexadecimal digits. A typical extended display is:



Notice the display for the hexadecimal digits for 10 through 15. Some are shown as upper case and some lower case: A b c d E F. As in the lecture notes we label each of the seven segments as shown:



When any one of the keys 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E or F is pressed, a 4-bit binary signal "wxyz" is generated, as shown in the table

Key	Binary representation				Segment	Fundamental
	w	**x**	**y**	**z**	**…**	**product**
0	0	0	0	0		
1	0	0	0	1		
2	0	0	1	0		
3	0	0	1	1		
4	0	1	0	0		
5	0	1	0	1		
6	0	1	1	0		
7	0	1	1	1		
8	1	0	0	0		
9	1	0	0	1		
A	1	0	1	0		
B	1	0	1	1		
C	1	1	0	0		
D	1	1	0	1		
E	1	1	1	0		
F	1	1	1	1		

For each of the seven segments there is a corresponding Boolean function and a circuit that has output 0 or 1 as the various keys are pressed. Denote these functions by a(w, x, y, z), b(w, x, y, z), …, g(w, x, y, z).

Your task concerns the two segments **d** and **g**, and is as follows. For **each** of the two functions **d**(w, x, y, z) and **g**(w, x, y, z):

(i) construct a truth table for the function (like the one shown above);

(ii) construct the corresponding Karnaugh map;

(iii) find a **minimal** sum of products for the function.

NOTE: Use the "simple" labelling on the Karnaugh maps and show your groups clearly.

[10 marks]